

A precise definition of the Standard Model

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Abstract

We declare that we are living in the quantum 4-dimensional Minkowski space-time with the force-fields gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ built-in from the very beginning. From this overall background, we see the lepton world, which has the symmetry characterized by $SU_L(2) \times U(1) \times SU_f(3)$ - the lepton world is also called "the atomic world". From the overall background, we also see the quark world, which experiences the well-known (123) symmetry, i.e., $SU_c(3) \times SU_L(2) \times U(1)$. The quark world is also called "the nuclear world".

The 3° K cosmic microwave background (CMB) in our Universe provides the evidence of that "the force-fields gauge-group structure was built-in from the very beginning". The CMB is almost uniform, to the level of one part in 10^5 , reflecting the massless of the photons.

The lepton world is dimensionless in the 4-dimensional Minkowski space-time. That is, all couplings are dimensionless. The quark world is also dimensionless in the 4-dimensional Minkowski space-time. Apart from the "ignition" term, the gauge and Higgs sector, i.e., the overall background, is also dimensionless. Thus, apart the "ignition" term, our world as a whole is dimensionless in the 4-dimensional Minkowski space-time - that is, it is the characteristic of the quantum 4-dimensional Minkowski space-time.

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1 Prelude

What is the Standard Model? It is a model that describes the behaviors of the point-like particles such as the electrons, the photons, the quarks, etc. What is the Standard Model of All Centuries [1]? We believe that the description of the point-like particles, the smallest units of matter in our Universe, on the basis of the Einstein's relativity principle and the quantum principle is so fundamental and also so complete and so consistent, that this Standard Model would stay there longer than the Newton's classic era.

The entries in the quantum 4-dimensional Minkowski space-time with force-fields gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ built-in from the very beginning mean the objects that have definitive properties both under the force group $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ and under the Lorentz group defining the quantum 4-dimensional Minkowski space-time.

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What precisely is the difference between "4-dimensional Minkowski space-time with the force-fields gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ built-in from the very beginning" and just "4-dimensional Minkowski space-time"? In the former, every object must have the designated-group assignments while, in the latter, it could be any 4-dimensional object in the Minkowski space-time. Is the adjective necessary in this context? We propose "yes" in the strictly mathematical sense.

We have to do all these with extra care, since "the Standard Model" is the beginning of everything. There exist basically no spoken rules in writing the Standard Model. In the Newton's classic era, different languages were developed for expressing ideas and things in a consistent and complete manner. In my search of the Standard Model, in the beginning it is not clear what I was searching for and after a period of successful searches I kept trying to spell out what the Standard Model really is. What is in the abstract of this paper, from the early version into the current one (completely revised), is a good illustrative example. It is for the Standard Model of All Centuries.

Back in the 20th Century, we didn't realized that there is something special for the complex scalar fields in the quantum 4-dimensional Minkowski space-time. A complex scalar field $\phi(x)$ is born to be self-repulsive, due to the $\lambda(\phi^\dagger\phi)^2$ interaction with the positive dimensionless coupling λ . With a negative λ , the system will collapse. So, if alone, *it cannot exist*.

For the two complex scalar fields, the attractive mutual interaction $-2\lambda(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)$ would be enough to overcome the self-repulsiveness of the two individual complex scalar fields. $\lambda(\phi^\dagger\phi)^2$ in fact writes the story for everything.

The story which we put forward above about the *nonexistence* of a single complex scalar field quite striking - whether it is right or not is still waiting for a clear-cut mathematical proof. Basically, λ cannot be negative since the system would collapse to the negative infinity. It cannot be zero since this would be meta-stable. We suspect $\lambda = \frac{1}{8}$ in our notations, but we should admit that it is still lack of rigorous proof.

Another important question is this: Why do we have the gauge bosons corresponding to the group $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ and, among these, why is only the photon massless? We think that we are not ready to answer a question like this - but eventually we would get enough hints for the final answer.

Among the force fields or gauge fields, most of the gauge fields, upon spontaneous symmetry breaking (SSB), become massive, including weak bosons and family gauge bosons, if the standard wisdom is assumed. That calls for the Standard-Model (SM) Higgs $\Phi(1, 2)$, the purely family Higgs $\Phi(3, 1)$, and the mixed family Higgs $\Phi(3, 2)$ - they interact attractively whenever possible, i.e., between $\Phi(1, 2)$ and $\Phi(3, 2)$, and between $\Phi(3, 1)$ and $\Phi(3, 2)$. (The two numbers/labels are referred to $SU_f(3)$ and $SU_L(2)$ - that is, triplets, doublets, or singlets.) Thus, these things provide the "background" of everything else. This is what we mean by the quantum 4-dimensional Minkowski space-time with the force-fields gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ imprinted at the very beginning.

So, if there would be only the SM Higgs $\Phi(1, 2)$, then *it cannot exist*. The self-repulsive interaction $\lambda(\phi^\dagger\phi)^2$ (with $\lambda > 0$) would make it from existence. As said earlier, $\lambda = 0$ would be meta-stable while $\lambda < 0$ would collapse. The question is why it is $\lambda = \frac{1}{8}$, and that should be determined *globally* by the quantum 4-dimensional Minkowski space-time, as this question doesn't arise if *not* the quantum 4-dimensional Minkowski space-time.

The algebra among the three Higgs $\Phi(1, 2)$, $\Phi(3, 2)$, and $\Phi(3, 1)$ arises *only* when it is in the quantum 4-dimensional Minkowski space-time. If the space-time differs from the 4-dimensional, the algebra simply doesn't apply. It is rather strange!!

The next question associated with the Higgs fields is to understand "the origin of mass" - a question that we have recently gained some understanding [2]. In that [2], we may set all the mass terms of the various Higgs to identically zero, except one spontaneous-symmetry-breaking (SSB) igniting term. All the mass terms are the results of this SSB, when switched on. Therefore, the "mass" is the result of SSB - a generalized Higgs mechanism. Thus, when the temperature is higher than a certain critical temperature, the notion of "mass" does not exist.

The set of the "various" Higgs includes the Standard-Model (SM) Higgs $\Phi(1, 2)$, the mixed family Higgs $\Phi(3, 2)$, and the pure family Higgs $\Phi(3, 1)$, where the first label refers to the group $SU_f(3)$ while the second the group $SU_L(2)$. The ignition could be on the pure family Higgs $\Phi(3, 1)$ [2], and it is clear that that it may not be on the SM Higgs $\Phi(1, 2)$.

These related Higgs, being the scalar fields, act as the systems of energies, self-interacting via dimensionless $\lambda(\phi^\dagger\phi)^2$ and interacting equivalently with other Higgs. When the temperature is low enough, it becomes the "mass" phase, or the phase in which the particles have masses.

2 The Lepton World and the Quark World

The overall background in our world is the quantum 4-dimensional Minkowski space-time with the force-field gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ imprinted at the very beginning. It sees the lepton world, of atomic sizes. It also sees the quark world, of much smaller nuclear sizes.

Thus, it is of importance to see that the $SU_L(2) \times U(1) \times SU_f(3)$ symmetry is realized in the lepton world, through the proposal [3] $((\nu_\tau, \tau)_L, (\nu_\mu, \mu)_L, (\nu_e, e)_L)$ (*columns*) ($\equiv \Psi(3, 2)$) as the $SU_f(3)$ triplet and $SU_L(2)$ doublet. It is also essential to complete the Standard Model [1] by working out the Higgs dynamics in detail [2]. Here it is important to realize the role of neutrino oscillations - it is the change of a neutrino in one generation (flavor) into that in another generation; or, we need to have the coupling $i\eta\bar{\Psi}_L(3, 2) \times \Psi_R(3, 1) \cdot \Phi(3, 2)$, exactly the coupling introduced by Hwang and Yan [3]. Then, it is clear that the mixed family Higgs $\Phi(3, 2)$ must be there. The remaining purely family Higgs $\Phi(3, 1)$ helps to complete the picture, so that the eight gauge bosons are massive in the $SU_f(3)$ family gauge theory [4]. [For consistency in our notations, it should be $\tilde{\Phi}(3, 2)$ in the paper of Hwang and Yan [3].]

With a complete Standard Model such as [1], we could address a few basic questions. After all, all "building blocks of matter" seem to be point-like particles (point-like Dirac particles if fermions), and vice versa. And nothing more. If quantum field theory (QFT) can describe the Nature, it should mean more - such as the various ultraviolet divergences, do they cancel out in some way? See below on tests for a "complete" theory.

Usually in an old textbook [5], the QCD chapter precedes the one on Glashow-Weinberg-Salam (GWS) electroweak theory, but we are talking about the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ Minkowski space-time and what happens in it. The so-called "basic units of motion"

are made up from quarks (of six flavors, of three colors, and of the two helicities) and leptons (of three generations and of the two helicities). We use these basic units (of motion) in writing down the lagrangian, etc. - the starting point of our formalism(s).

If we look at the basic units of motion as compared to the original particle, i.e. the electron, the starting basic units are all "point-like" Dirac particles. Dirac invented the Dirac equation for the electron eighty years ago and surprisingly enough these "point-like" Dirac particles are the basic units of the Standard Model. Thus, we call it "Dirac Similarity Principle" - a salute to Dirac; a triumph to mathematics. Our world could indeed be described by the proper mathematics. The proper mathematical language may be the renormalizable quantum field theory, as advocated in this paper.

For the lepton world or the quark world, the story is fixed if the so-called "gauge-invariant derivative", i.e. D_μ in the kinetic-energy term $-\bar{\Psi}\gamma_\mu D_\mu\Psi$, is given for a given basic unit, one on one [5].

For the lepton world, we introduce the family triplet, $(\nu_\tau^R, \nu_\mu^R, \nu_e^R)$ (column), under $SU_f(3)$. Since the minimal Standard Model does not see the right-handed neutrinos, it would be a natural way to make an extension of the minimal Standard Model. Or, we have, for $(\nu_\tau^R, \nu_\mu^R, \nu_e^R)$,

$$D_\mu = \partial_\mu - i\kappa \frac{\bar{\lambda}^a}{2} F_\mu^a. \quad (1)$$

and, for the left-handed $SU_f(3)$ -triplet and $SU_L(2)$ -doublet $((\nu_\tau^L, \tau^L), (\nu_\mu^L, \mu^L), (\nu_e^L, e^L))$ (all columns),

$$D_\mu = \partial_\mu - i\kappa \frac{\bar{\lambda}^a}{2} F_\mu^a - ig \frac{\vec{\tau}}{2} \cdot \vec{A}_\mu + i\frac{1}{2}g' B_\mu. \quad (2)$$

The right-handed charged leptons form the triplet $\Psi_R^C(3,1)$ under $SU_f(3)$, since it were singlets their common factor $\bar{\Psi}_L(\bar{3},2)\Psi_R(1,1)\Phi(3,2)$ for the mass terms would involve the cross terms such as $\mu \rightarrow e$.

The neutrino mass term assumes a new form [3]:

$$i\frac{h}{2}\bar{\Psi}_L(3,2) \times \Psi_R(3,1) \cdot \tilde{\Phi}(3,2) + h.c., \quad (3)$$

where $\Psi(3,i)$ are the neutrino triplet just mentioned above (with the first label for $SU_f(3)$ and the second for $SU_L(2)$). The cross (curl) product is somewhat new [4], referring to the singlet combination of three triplets in $SU(3)$. The Higgs field $\tilde{\Phi}(3,2)$ is new in this effort, because it carries some nontrivial $SU_L(2)$ charge.

Note that, for charged leptons, the Standard-Model choice is $\bar{\Psi}(\bar{3},2)\Psi_R^C(3,1)\Phi(1,2)+c.c.$, which gives three leptons an equal mass. But, in view of that if (ϕ_1, ϕ_2) is an $SU(2)$ doublet then $(\phi_2^\dagger, -\phi_1^\dagger)$ is another doublet, we could form $\tilde{\Phi}(3,2)$ from the doublet-triplet $\Phi(3,2)$. The notations in $\Phi(1,2)$, $\Phi(3,2)$, and $\Phi(3,1)$ should be consistent and thus the $\tilde{\Phi}(3,2)$, used in the above equation, should have the tilde operation, for the consistency in notations.

So, we have [1]

$$i\frac{h^C}{2}\bar{\Psi}_L(3,2) \times \Psi_R^C(3,1) \cdot \Phi(3,2) + h.c., \quad (4)$$

which gives rise to the imaginary off-diagonal (hermitian) elements in the 3×3 mass matrix, so removing the equal masses of the charged leptons. Note that the couplings h , h^C , and κ all are dimensionless.

The expressions for neutrino oscillations and the off-diagonal mass term are in $i\epsilon_{abc}$, or curl-dot, product - it is allowed for $SU(3)$. Note that such coupling has nothing to do with the kinetic-energy term of the particle, though the coupling h (and h^c) might be related to the gauge coupling κ .

We now turn our attention to the quark world, which our special gauge-group Minkowski space-time supports. Thus, we have, for the up-type right-handed quarks u_R , c_R , and t_R ,

$$D_\mu = \partial_\mu - ig_c \frac{\lambda^a}{2} G_\mu^a - i \frac{2}{3} g' B_\mu, \quad (5)$$

and, for the rotated down-type right-handed quarks d'_R , s'_R , and b'_R ,

$$D_\mu = \partial_\mu - ig_c \frac{\lambda^a}{2} G_\mu^a - i \left(-\frac{1}{3}\right) g' B_\mu. \quad (6)$$

On the other hand, we have, for the $SU_L(2)$ quark doublets,

$$D_\mu = \partial_\mu - ig_c \frac{\lambda^a}{2} G_\mu^a - ig \frac{\vec{\tau}}{2} \cdot \vec{A}_\mu - i \frac{1}{6} g' B_\mu. \quad (7)$$

There are the standard way to generate mass for the various quarks. For these quarks, we use the "old-fashion" way as in the Standard Model, since quarks do not couple to the family Higgs fields. We have, for the generation of the various quark masses,

$$\begin{aligned} & G_1 \bar{U}_L(1,2) u_R \tilde{\Phi}(1,2) + G'_1 \bar{U}_L(1,2) d'_R \Phi(1,2) + h.c. + \\ & G_2 \bar{C}_L(1,2) c_R \tilde{\Phi}(1,2) + G'_2 \bar{C}_L(1,2) s'_R \Phi(1,2) + h.c. + \\ & G_3 \bar{T}_L(1,2) t_R \tilde{\Phi}(1,2) + G'_3 \bar{T}_L(1,2) b'_R \Phi(1,2) + h.c., \end{aligned} \quad (8)$$

with the tilde's defined as before.

Again, all the couplings in the quark world are dimensionless in the 4-dimensional Minkowski space-time. Surprisingly, the natural scale for the quark world is of fermi scales, which is five orders smaller than the natural scale of the lepton world, of atomic scales.

It might be essential to realize that the dimensionless couplings g_c , g , g' (hence α), and κ (in the strengths of the fundamental interactions) and the dimensionless mass parameters h , h^C , $G_{1,2,3}$, $G'_{1,2,3}$ (in describing the masses of point-like particles) have a complete equal status in the philosophy of concepts.

The overall background, i.e., the quantum 4-dimensional Minkowski space-time with the force-fields gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ built-in from the outset, supports the "dimensionless" lepton world and it also supports the "dimensionless" quark world. It seems that there might be many elegant stories associated with the Standard Model (of all centuries) [1].

3 The Overall Background

We should present our reasonings which lead to the formulation of "The Origin of Mass" [2]. It stresses that, before the spontaneous symmetry breaking (SSB), the Standard Model does not contain any parameter that is pertaining to "mass", but, after the SSB, all particles

in the Standard Model acquire the mass terms as it should - a way to explain "the origin of mass". In this way, we sort of tie "the origin of mass" to the effects of the SSB, or the generalized Higgs mechanism.

It is amusing to observe that it is so easy, by construction, to have the complex scalar fields but, among the building blocks of matter, the complex scalar fields are so rare. It seems to be much harder for two complex scalar fields in co-existence, as though they would be mutually "repulsive". Only if they belong to the "same" family, they might be mutually attractive.

In the 4-dimensional Minkowski space-time, it is an amusing fact for the complex scalar field that the dimensionless interaction $\lambda(\phi^\dagger\phi)^2$ exists - we don't know how to determine the dimensionless λ ; this might have to do with the 4-dimensional nature and maybe more. The determination of λ , that should be done *a priori* in the Standard Model, poses an important conceptional question.

The reason that we try to write together a force-field Minkowski space-time is that when put together the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ Minkowski space-time (that is already specific enough) the complex scalar field ϕ in this space should have a specific λ in the dimensionless interaction $\lambda(\phi^\dagger\phi)^2$. If we agree that a specific λ is needed, then there is the universal λ for the various complex scalar fields allowed in this force-field Minkowski space-time. The complex scalar field(s) should have the existence *a priori*. [These arguments sound fairly philosophical and logical, but they are needed for clarification.]

A complex scalar field in our space-time has the dimensionless coupling:

$$V(x) = \lambda(\phi^\dagger(x)\phi(x))^2. \quad (9)$$

The space-time integral of $L = T - V$ gives the action. In our 4-dimensional Minkowski space-time, we find that $\lambda = \frac{1}{8}$ numerically. This number should come out topologically (after the normalizations of the various fields in a given space [5]), although, at this point, we don't know why this is the case.

If there are more than a complex scalar field, we should have

$$V(x) = \lambda\{(\phi_a^\dagger\phi_a)^2 + (\phi_b^\dagger\phi_b)^2 + \dots\}. \quad (10)$$

There should be only one λ .

For the two related complex fields, we propose to write

$$V(x) = \lambda\{(\phi_a^\dagger\phi_a + \phi_b^\dagger\phi_b)^2 - 4(\phi_a^\dagger\phi_b) \cdot (\phi_b^\dagger\phi_a)\}, \quad (11)$$

to signify the mutual attraction on top of the universal repulsive interactions.

Now we return to the Standard Model of All Centuries [1]. We have the Standard-Model Higgs $\Phi(1,2)$, the purely family Higgs $\Phi(3,1)$, and the mixed family Higgs $\Phi(3,2)$, with the first label for $SU_f(3)$ and the second for $SU_L(2)$. We need another triplet $\Phi(3,1)$ since all eight family gauge bosons are massive [4].

It is clear that $\Phi(1,2)$ would interact with $\Phi(3,2)$ while $\Phi(3,1)$ would also interact with $\Phi(3,2)$. These interactions should be attractive to explain why they are showing up together.

We try to come back to the ground-zero point. Thus, we may write down the general terms for potentials among the three Higgs fields, subject to (1) that they are renormalizable, and (2) that symmetries are only broken spontaneously (the Higgs or induced Higgs mechanism). We have [1, 5]

$$V = V_{SM} + V_1 + V_2 + V_3, \quad (12)$$

$$V_{SM} = \mu^2 \Phi^\dagger(1, 2) \Phi(1, 2) + \lambda (\Phi^\dagger(1, 2) \Phi(1, 2))^2 \quad (13)$$

$$\begin{aligned} V_1 = & M^2 \Phi^\dagger(\bar{3}, 2) \Phi(3, 2) + \lambda_1 (\Phi^\dagger(\bar{3}, 2) \Phi(3, 2))^2 \\ & + \epsilon_1 (\Phi^\dagger(\bar{3}, 2) \Phi(3, 2)) (\Phi^\dagger(1, 2) \Phi(1, 2)) + \eta_1 (\Phi^\dagger(\bar{3}, 2) \Phi(1, 2)) (\Phi^\dagger(1, 2) \Phi(3, 2)) \\ & + \epsilon_2 (\Phi^\dagger(\bar{3}, 2) \Phi(3, 2)) (\Phi^\dagger(\bar{3}, 1) \Phi(3, 1)) + \eta_2 (\Phi^\dagger(\bar{3}, 2) \Phi(3, 1)) (\Phi^\dagger(\bar{3}, 1) \Phi(3, 2)) \\ & + (\delta_1 i \Phi^\dagger(3, 2) \times \Phi(3, 2) \cdot \Phi^\dagger(3, 1) + h.c.), \end{aligned} \quad (14)$$

$$\begin{aligned} V_2 = & \mu_2^2 \Phi^\dagger(\bar{3}, 1) \Phi(3, 1) + \lambda_2 (\Phi^\dagger(\bar{3}, 1) \Phi(3, 1))^2 \\ & + (\delta_2 i \Phi^\dagger(3, 1) \cdot \Phi(3, 1) \times \Phi^\dagger(3, 1) + h.c.) \\ & + \lambda_2' \Phi^\dagger(\bar{3}, 1) \Phi(3, 1) \Phi^\dagger(1, 2) \Phi(1, 2), \end{aligned} \quad (15)$$

$$\begin{aligned} V_3 = & (\delta_3 i \Phi^\dagger(3, 2) \cdot \Phi(3, 2) \times (\Phi^\dagger(1, 2) \Phi(3, 2)) + h.c.) \\ & + (\delta_4 i (\Phi^\dagger(3, 2) \Phi(1, 2)) \cdot \Phi^\dagger(3, 1) \times \Phi(3, 1) + h.c.) \\ & + \eta_3 (\Phi^\dagger(\bar{3}, 2) \Phi(1, 2) \Phi(3, 1) + c.c.). \end{aligned} \quad (16)$$

In doing the renormalization analysis of the three Higgs fields, we realize that even if we start from the well-motivated lagrangian such as Eq. (11), it might spill over to the more generalized lagrangian such as Eqs. (12)-(16).

We might pay special attention to the so-called "U-gauge" (unitary gauge). In the U-gauge, every particle is a real particle (not a ghost). We find it to be useful in the analysis of the situation with the spontaneous symmetry breaking (SSB). For the quantum 4-dimensional Minkowski space-time with the force-fields gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ built-in from the very beginning (i.e., the overall background), we have, in the U-gauge, W^\pm , Z^0 , and eight massive family gauge bosons, one Standard-Model Higgs and four neutral family Higgs (three mixed plus one pure).

Thus, we choose to have, in the U-gauge, as in [2],

$$\Phi(1, 2) = (0, \frac{1}{\sqrt{2}}(v + \eta)), \quad \Phi^0(3, 2) = \frac{1}{\sqrt{2}}(u_1 + \eta'_1, u_2 + \eta'_2, u_3 + \eta'_3), \quad \Phi(3, 1) = \frac{1}{\sqrt{2}}(w + \eta', 0, 0), \quad (17)$$

all in columns. The five components of the complex triplet $\Phi(3, 1)$ get absorbed by the $SU_f(3)$ family gauge bosons and the neutral part of $\Phi(3, 2)$ has three real parts left - together making all eight family gauge bosons massive.

Before the mixing, the masses of the various Higgs are given by, for Eqs. (12)-(16),

$$\begin{aligned} \eta : & \quad (\mu^2/\lambda) + \frac{1}{4}(\epsilon_1 + \eta_1)u_i u_i + \frac{\lambda_2'}{4}w^2, \\ \eta' : & \quad (\mu_2^2/\lambda_2) + \frac{\epsilon_2}{4}u_i u_i + \frac{\eta_2}{4}u_1^2 + \frac{\lambda_2'}{4}v^2, \\ \eta'_1 : & \quad M^2 + \frac{1}{4}(\epsilon_1 + \eta_1)v^2 + \frac{1}{4}(\epsilon_2 + \eta_2)w^2 + (\lambda_1 - term), \\ \eta'_{2,3} : & \quad M^2 + \frac{1}{4}(\epsilon_1 + \eta_1)v^2 + \frac{\epsilon_2}{4}w^2 + (\lambda_1 - term), \end{aligned}$$

$$\begin{aligned}
\phi_1 : & \quad M^2 + \frac{1}{2}\epsilon_1 v^2 + \frac{1}{2}\epsilon_2 w^2 + \frac{1}{2}\eta_2 w^2 + \frac{\lambda_1}{2}u_i u_i, \\
\phi_{2,3} : & \quad M^2 + \frac{1}{2}\epsilon_1 v^2 + \frac{1}{2}\epsilon_2 w^2 + \frac{\lambda_1}{2}u_i u_i.
\end{aligned} \tag{18}$$

The mixing term looks like, apart from some common factor:

$$2(\epsilon_1 + \eta_1)u_i \eta'_i v \eta + 2\epsilon_2 u_i \eta'_i w \eta' + 2\eta_2 u_1 \eta'_1 w \eta' + 2\lambda'_2 w \eta' v \eta. \tag{19}$$

And we also neglect the mixing (and the mixing inside $\eta'_{1,2,3}$). To understand "the origin of mass", we would drop out all "mass" terms to begin with.

To understand the origin of mass [2], we find that the ignition term would better be in the purely family sector, i.e., the μ_2^2 term. When $\mu_2^2 = 0$, the $\Phi(3, 2)$ is equally partitioned between $\Phi(1, 2)$ and $\Phi(3, 1)$.

It is easy to see that only one SSB-driving term is enough for all the three Higgs fields – there may be several SSB's for the neutral fields – in our case, it works for all of them. SSB for one Higgs but is driven by other Higgs – a unique feature for the complex scalar fields. Or, we have [2]

$$\begin{aligned}
V_{Higgs} = & \mu_2^2 \Phi^\dagger(3, 1)\Phi(3, 1) + \lambda(\Phi^\dagger(1, 2)\Phi(1, 2) + \cos\theta_P \Phi^\dagger(3, 2)\Phi(3, 2))^2 \\
& + \lambda(-4\cos\theta_P)(\Phi^\dagger(\bar{3}, 2)\Phi(1, 2))(\Phi^\dagger(1, 2)\Phi(3, 2)) \\
& + \lambda(\Phi^\dagger(3, 1)\Phi(3, 1) + \sin\theta_P \Phi^\dagger(3, 2)\Phi(3, 2))^2 \\
& + \lambda(-4\sin\theta_P)(\Phi^\dagger(\bar{3}, 2)\Phi(3, 1))(\Phi^\dagger(3, 1)\Phi(3, 2)).
\end{aligned} \tag{20}$$

These are two perfect squares minus the other extremes, to guarantee the positive definiteness, when the minus μ_2^2 was left out. (θ_P may be referred to as "Pauchy's angle".) ϵ_1 , η_1 , and ϵ_2 , η_2 are expressed in λ , a great simplification. Note that we only include the interference terms between those involving the same group, $SU_f(3)$ or $SU_L(2)$; thus $\lambda'_2 = 0$.

From the expressions of $u_i u_i$ and v^2 , we obtain

$$v^2(3\cos^2\theta_P - 1) = \sin\theta_P \cos\theta_P w^2. \tag{21}$$

And the SSB-driven η' yields

$$w^2(1 - 2\sin^2\theta_P) = -\frac{\mu_2^2}{\lambda} + (\sin 2\theta_P - \tan\theta_P)v^2. \tag{22}$$

These two equations show that it is necessary to have the driving term, since $\mu_2^2 = 0$ implies that everything is zero. Also, $\theta = 45^\circ$ is the (lower) limit.

The mass squared of the SM Higgs η is $2\lambda\cos\theta_P u_i u_i$, as known to be $(125 \text{ GeV})^2$. The famous v^2 is the number divided by 2λ , or $(125 \text{ GeV})^2/(2\lambda)$. Using PDG's for e , $\sin^2\theta_W$, and the W -mass [10], we find $v^2 = 255 \text{ GeV}$. So, $\lambda = \frac{1}{8}$, a simple model indeed.

The ratio of the VEV to its Higgs mass is determined by 2λ , whether the channel is not ignited or not. We might choose the channel of η' (the purely family Higgs) or that of η (the SM Higgs) as the ignition channel, but three Higgs channels have different labels. The three Lorentz-invariant scalar fields have different internal structures – an amusing question for further investigation.

The mass squared of η' is $-2(\mu_2^2 - \sin\theta_P u_1^2 + \sin\theta_P(u_2^2 + u_3^2))$. The other condensates are $u_1^2 = \cos\theta_P v^2 + \sin\theta_P w^2$ and $u_{2,3}^2 = \cos\theta_P v^2 - \sin\theta_P w^2$ while the mass squared of η'_1 is $2\lambda u_1^2$, those of $\eta'_{2,3}$ be $2\lambda u_{2,3}^2$. The mixings among η'_i themselves are neglected in this paper.

There is no SSB for the charged Higgs $\Phi^+(3, 2)$. The mass squared of ϕ_1 is $\lambda(\cos\theta_P v^2 - \sin\theta_P w^2) + \frac{\lambda}{2} u_i u_i$ while $\phi_{2,3}$ be $\lambda(\cos\theta_P v^2 + \sin\theta_P w^2) + \frac{\lambda}{2} u_i u_i$. (Note that a factor of $\frac{1}{2}$ appears in the kinetic and mass terms when we simplify from the complex case to that of the real field; see Ch. 13 of the Wu-Hwang book [5].)

A further look of these equations tells that $3\cos^2\theta_P - 1 > 0$ and $2\sin^2\theta_P - 1 > 0$. A narrow range of θ_P is allowed (greater than 45° while less than 57.4° , which is determined by the group structure). For illustration, let us choose $\cos\theta_P = 0.6$ and work out the numbers as follows: (Note that $\lambda = \frac{1}{8}$ is used.)

$$\begin{aligned}
6w^2 &= v^2, & -\mu_2^2/\lambda &= 0.32v^2; \\
\eta : & 2\lambda\cos\theta_0 u_i u_i = (125 \text{ GeV})^2, & v^2 &= (250 \text{ GeV})^2; \\
\eta' : & mass^2 = (51.03 \text{ GeV})^2, & w^2 &= v^2/6; \\
\eta'_1 : & mass^2 = (107 \text{ GeV})^2, & u_1^2 &= 0.7333v^2; \\
\eta'_{2,3} : & mass^2 = (85.4 \text{ GeV})^2, & u_{2,3} &= 0.4667v^2; \\
\phi_1 : & mass = 100.8 \text{ GeV}; & \phi_{2,3} : & mass = 110.6 \text{ GeV}.
\end{aligned} \tag{23}$$

All numbers appear to be reasonable. In the above, $\cos\theta_P$ is the only free parameter until one of the family Higgs particles $\eta'_{1,2,3}$ and η' is found experimentally. Since the new objects need to be accessed in the lepton world, it would be a challenge for our experimental colleagues.

As a footnote, our Standard Model predicts that the mass of the SM Higgs η is a half of the vacuum expectation value v - a prediction in the origin of mass [2].

As for the range of validity, $\frac{1}{3} \leq \cos^2\theta_P \leq \frac{1}{2}$. The first limit refers to $w^2 = 0$ while the second for $\mu_2^2 = 0$.

We may fix up the various couplings, using our common senses. The cross-dot products would be similar to κ , the basic coupling of the family gauge bosons. The electroweak coupling g is 0.6300 while the strong QCD coupling $g_s = 3.545$; my first guess for κ would be about 0.1. The masses of the family gauge bosons would be estimated by using $\frac{1}{2}\kappa \cdot w$, so slightly less than 10 GeV . (In the numerical example with $\cos\theta_P = 0.6$, we have $6w^2 = v^2$ or $w = 102 \text{ GeV}$. This gives $m = 5 \text{ GeV}$ as the estimate.) So, the range of the family forces, existing in the lepton world, would be 0.02 fermi .

In [2], the term that ignites the SSB is chosen to be with η' , the purely family Higgs. This in turn ignites EW SSB and others. It explains the origin of all the masses, in terms of the spontaneous symmetry breaking (SSB). SSB in $\Phi(3, 2)$ is driven by $\Phi(3, 1)$, while SSB in $\Phi(1, 2)$ from the driven SSB by $\Phi(3, 2)$, as well. The different, but related, scalar fields can accomplish so much, to our surprise.

We note that, at the Lagrangian level, the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ gauge symmetry is protected but the symmetry is violated via spontaneous symmetry breaking (via the Higgs mechanisms).

We iterate that the mathematics of the three neutral Higgs, $\Phi(1, 2)$ (Standard-Model Higgs), $\Phi(3, 1)$ (purely family Higgs), and $\Phi^0(3, 2)$ (mixed family Higgs), subject to the renormalizability (up to the fourth power), turns to be rather rich. In our earlier work

regarding the "colored Higgs mechanism" [6], we show how the eight gauge bosons in the $SU(3)$ gauge theory become massive using two complex scalar triplet fields (with the resultant four real Higgs fields), with a lot of choices. We suspect that, even within QCD, there might be some elegant choice of "colored" Higgs, or there must be a good reason for massless gluons.

4 The Standard Model as a complete theory

We declare that we are living in the quantum 4-dimensional Minkowski space-time with the force-fields gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ built-in from the very beginning. This "overall background" can see the lepton world, of atomic sizes, that has the $SU_L(2) \times U(1) \times SU_f(3)$ symmetry, or the other (123) symmetry. It also can see the quark world of much smaller nuclear sizes, which possesses the $SU_c(3) \times SU_L(2) \times U(1)$ symmetry, or the standard (123) symmetry.

We make this declaration so that we spell out an end to Newton's classic era of four centuries. It is fundamental to recognize that we are living in a "new" world, that is different from the classic Newton's world.

The lepton world is dimensionless in the 4-dimensional Minkowski space-time. The quark world is also dimensionless in the 4-dimensional Minkowski space-time. Except the SSB "ignition" term, the overall background is also dimensionless in the 4-dimensional Minkowski space-time. "Dimensionless in the 4-dimensional Minkowski space-time" means that it is determined *globally* by the quantum 4-dimensional Minkowski space-time.

To be precise, the lepton world behaves well in the quantum 4-dimensional Minkowski space-time built-in with the force-fields gauge-group structure $SU_L(2) \times U(1) \times SU_f(3)$. Meanwhile, the quark world behaves well in the quantum 4-dimensional Minkowski space-time built-in with the force-fields gauge-group structure $SU_c(3) \times SU_L(2) \times U(1)$. All the couplings are dimensionless and, thus, they are determined *globally* by the quantum 4-dimensional Minkowski space-time.

Can we implement some methodology of ultraviolet divergences such that everything is determined *globally* by the quantum 4-dimensional Minkowski space-time with some force-fields gauge-group structure built-in from the very beginning? As we shall see, it is rather difficult to obtain clear-cut answers to these questions.

The lepton world respects the $SU_L(2) \times U(1) \times SU_f(3)$ symmetry such that, in principle, the divergences in the lepton world should "operate" among themselves. Similar cases happen for the quark world. Likewise, those in the overall background should also "operate" among themselves. The mix-ups among the different sectors occur so easily if we go to higher "orders".

Let's try to think following a specific example. We consider the the self-energy of the Standard-Model (SM) Higgs $\Phi(1,2)$, depicted by Figs. 1. It is obvious that there are mix-ups among the different sectors.

Specifically, for the sake of simplicity, we are focusing our attention only on the ultraviolet divergences of highest order in one loop.

Hence we could only begin the analysis of "one" such question while leaving the heavy burdens to the others. In the textbook, e.g., Ch. 10 of the Wu-Hwang book [5] on the

ultraviolet divergences in QED, the divergences are there, but QED is only part of the theory, including the leading-order calculation related to $g - 2$. We propose that many issues could be examined in a theory in the quantum 4-dimensional Minkowski space-time with the force-fields gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ built-in from the very beginning.

Although one electron self-energy diagram shown in Ch. 10 of the Wu-Hwang book [5] is infinite, this is true in QED but QED is *only* part of the Standard Model. Maybe all of the self-energy diagrams of similar type could add up to a finite number. The fact that the QED-part is infinite is the symptom of that QED is an incomplete theory. (One can argue this way.) In a complete theory, we should have everything, and nothing more - if there is still some infinities, then there is something more; this is a "new" way of searching for new things, i.e., the hard way, to find something new.

In fact, we would like to show that the U-gauge and the dimensional regularization offer us the *machinery* to handle these infinities. Staying in the U-gauge means that we are treating "physical particles". Dimensional regularization means that the results in all dimensions are included, including finite results in fractional dimensions.

But, unfortunately, the dimensional regularization *does not care about the causality $i\epsilon$ requirements* so that the results are only telling us something, and they may not be the real final numbers. In fact, the answers which we obtain in the dimensional regularization scheme may not be *true* answers (under the casual $i\epsilon$ prescription), since in the dimensional regularization they have their own definition of the fractional-dimensional integrals.

On the negative side, in the dimensional regularization scheme, one tries to define the results for the fractional dimensions and, by continuation, obtain the results for the integral dimensions. Unlike the Pauli-Villars regularization (e.g., Ch. 10 of [5]), one tries to manipulate the causality $i\epsilon$ requirement in obtaining the final results.

We try to follow the details of [2] in discussing the ultraviolet divergences of the quadratic order of the self-energy of the SM Higgs, but adding some new points.

$$\begin{aligned}
& \frac{\eta(\phi^0(1,2))}{(\mu^2 < 0)} + \lambda \text{ (circle)} + \epsilon_1 \text{ (dashed circle)} \\
& + \lambda'_2 \text{ (dashed circle)} + \eta_1 \text{ (dashed circle with cross)} \\
& + \delta_3 i \text{ (dashed circle with cross)} + \eta_3 \text{ (dashed circle with cross)} \\
& + \lambda^2 \text{ (double circle)} + \lambda^3 \text{ (triple circle)} + \dots \\
& + \dots \text{ (similar connected loops)}
\end{aligned}$$

Figure 1: The within-Higgs diagrams for the Standard-Model Higgs $\Phi(1,2)$.

In Fig. 1, the wave-function renormalization of the Standard-Model Higgs $\Phi(1,2)$ is

shown, for simplicity, in the U-gauge in the absence of Dirac fermions. The lowest-order loop diagrams, from the above interaction lagrangian, are shown from 1(b) [in λ] to 1(g) [in η_3], where the first five are of quadratic divergence while the last one of logarithmic divergence. The higher-order connected loop diagrams, many of them and also of quadratic divergence, are also troublesome and should be dealt with at the same time. We will discuss the cases of the worst divergent, i.e., the quadratic divergent, in what follows.

Using dimensional regularization (i.e. the appendix of Ch. 10, the Wu-Hwang book, Ref. [5]), we could write down the one-loop results.

We try to use one explicit example to illustrate our point related to the infinities - the quadratic divergences of the wave function of the SM Higgs η . Here, in Fig. 1, we try to show only the Higgs sectors themselves; in a complete Standard Model, the (Dirac) fermion loop diagrams, and those with gauge bosons, also present divergences of quadratic order and should be dealt with simultaneously.

As said earlier, we know that the formulae in dimensional regularization give us some things, *apart from the $-i\epsilon$ prescription*, and that it "works" in the U-gauge. For example, the Z^0 - *boson* loop for Fig. 1 would give us the vanishing result - so, it does not bother us.

In details, the coupling of the SM Higgs is ([5], e.g., Wu/Hwang, Ch. 13, Ref. [5])

$$-\frac{1}{8}(v^2 + 2v\eta + \eta^2)\{2g^2 W_\mu^+ W_\mu^- + [g^2 + (g')^2]Z_\mu^0 Z_\mu^0\}, \quad (24)$$

which gives rise to, to the first order, the one-loop W^\pm or Z^0 diagram. To evaluate them, we use the propagator in the U-gauge (see the appendix of Ch. 13, Ref. [5]) and the formulae in the dimensional regularization (see the appendix of Ch. 10, Ref. [5]). They cancel between two terms for each diagram.

We proceed to examine those diagrams in Fig. 1 which are "simple" quadratically divergent - those at the one-loop order. These are among the various Higgs.

The one-loop diagrams involving the quark (or charged lepton), when simplified, are sums of quadratic and logarithmic divergences.

Using dimensional regularization (i.e. the appendix of Ch. 10, the Wu-Hwang book, [5]), we obtain the one-loop and quadratic-divergence results as follows. In the dimensional regularization, the factor $\Gamma(1 - \frac{n}{2})$ stands for where the quadratic divergence appears. Maybe the fractional dimensions, which are represented as finite numbers, could get some meaning, but we have to remember that, as a drawback, we bypass the $-i\epsilon$ in the propagators.

$$\begin{aligned} & -4 \cdot \frac{n}{2} \cdot (S_q + S_{c.l.}) \Gamma(1 - \frac{n}{2}) \\ & + \{3\lambda m^2(\eta) + \frac{\epsilon_1}{2} \sum_i m^2(\eta'_i) + \epsilon_1 \sum_i m^2(\phi_i) \\ & + \frac{\lambda'}{2} m^2(\eta') + \frac{n}{2} \sum_i m^2(\eta'_i)\} \Gamma(1 - \frac{n}{2}); \end{aligned} \quad (25)$$

$$\begin{aligned} S_q &= \sum_{quarks} 3 \cdot G_i^2 \cdot (m_i^2 - \frac{1}{6} m^2(\eta)), \\ S_{c.l.} &= 3 \cdot \bar{G}_l^2 \cdot (\bar{m}_l^2 - \frac{1}{6} m^2(\eta)). \end{aligned} \quad (26)$$

Or, using the Standard Model of the paper, we have

$$\begin{aligned} & -4 \cdot \frac{n}{2} \cdot (S_q + S_{c.l.}) \Gamma(1 - \frac{n}{2}) \\ & + \{\lambda(3m^2(\eta) - \cos\theta_P \sum_i m^2(\eta'_i) + 2\cos\theta_P \sum_i m^2(\phi_i))\} \Gamma(1 - \frac{n}{2}). \end{aligned} \quad (27)$$

Here we don't equal the sum to anything since, as we said before, the dimensional regularization scheme don't take care of the causality $i\epsilon$ prescription.

Maybe we could proceed in the following way. After removing the divergent $\Gamma(1 - \frac{n}{2})$ parts, we might propose to use

$$\begin{aligned} & -4 \cdot \frac{n}{2} \cdot (S_q + S_{c.l.}) \\ & + \{\lambda(3m^2(\eta) - \cos\theta_P \sum_i m^2(\eta'_i) + 2\cos\theta_P \sum_i m^2(\phi_i))\} \approx 0; \end{aligned} \quad (28)$$

or something finite.

Here we would be happy to use an equal sign instead of the approximate sign, if the causality $-i\epsilon$ prescription could be used instead.

There are a few general characteristics: (1) In the contributions from quarks and from charged leptons, the mass of the SM Higgs enters (as external momentum squared). This makes all contributions equivalent in some sense. (2) We assume that the quarks enter in the theory in the SM way - if we examine the theory closely, there might still be some room for colored Higgs mechanism [6]. In other words, we are not sure that the "identity" in this SM Higgs would hold out; instead, the identity in the case of pure Higgs $\Phi(3, 1)$, or that for $\Phi(3, 2)$, has better reasons to hold out. Or, in light of the fact that it is difficult to rule out the colored Higgs mechanism in the quark world.

In deriving the above equations, the coefficients of $\Gamma(1 - \frac{n}{2})$ are the coefficients of quadratic divergences while those of $\Gamma(2 - \frac{n}{2})$ are the coefficients of logarithmic divergences - for the latter, divergence is less severe and the contributions could be everywhere; the treatment is far more complicated.

As mentioned in [2], the diagrams which are of multiple quadratic divergence are troublesome since the series could be blown-up, of $2n$ -th divergence with $n \rightarrow \infty$. Mathematically, we should avoid such terms by all means. This is the requirement beyond the naive renormalizability.

We are hinting that we have to study the mathematics of divergences; they are there, because of the uncountably infinite degrees of freedom and other reasons, and there are regularities to be discovered [7]. So, is the Standard Model a complete theory?

The result for $\Phi(1, 2)$ in the one-loop result certainly gives rise to an "approximate" formula for the masses and the couplings - they are "approximate" because the cancelation could be modified by terms in the higher-loop orders. But it should be approximate - that is why it is worthwhile looking for them.

According to dimensional-regularization results, the three-loop diagram gives rise to (quadratic divergence) \times (logarithmic divergence), and so on. They have to organized differently. One simple way out is that they cancel completely in their own group, such as all the four-loop diagrams. In any event, these divergences could be systematically analyzed to display what is going on.

Maybe we have said too much about the dimensional-regularization results, we in fact are suspicious that these results may not make any sense since they ignore the causality $-i\epsilon$ requirement but the causality $-i\epsilon$ requirement is a must. Let's look at the fermion-loop diagram for the Higgs $\Phi(1, 2)$, which is given by the previous $S_{c.l.}$ term in the dimensional regularization scheme.

$$\begin{aligned}
& I(k^2) \\
&= \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \left\{ \frac{m+i\gamma \cdot p}{m^2+p^2-i\epsilon} \frac{m-i\gamma \cdot (p+k)}{m^2+(p+k)^2-i\epsilon} (1-\gamma_5) \right\} \\
&= 4 \int \frac{m^2+p \cdot (p+k)}{(m^2+p^2-i\epsilon)(m^2+(p+k)^2-i\epsilon)} \\
&\equiv (I(k^2) - I(0)) + I(0).
\end{aligned} \tag{29}$$

Or, it can be represented as a Taylor's series.

In $I(0)$, it becomes a simple pole in the k_0 integration. In the next Taylor's expansion, integration by parts could help us to convert the double pole to a simple pole. So, the math tricks could make everything transparent to us.

But, in light to the infinities, it is not obvious on how to proceed. What we are doing is a little similar to the Pauli-Villars regularization scheme, e.g., Ch. 10 of the Wu-Hwang book [5], though we didn't work out infinities. The dimensional regularization scheme [5] ignores the causality $i\epsilon$ requirement; in our opinions, we don't know what we are doing.

If we believe that all point-like particles are there and that the Standard Model describes them so well, then some sort of cancelation conjecture or hypothesis might hold, first for the simple quadratic divergences of a certain Higgs, and then other divergences. Of course some theorem(s) of "global" type may be needed. The "new" marriage of mathematics and physics is needed for the ultimate breakthrough. Infinities in our language should not discourage us since the degrees of freedom in quantum fields are more than enumerable infinite.

5 Other Important Observations

It is very strange that our overall background is the quantum 4-dimensional Minkowski space-time with the force-fields gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ built-in from the very beginning. The uniform $3^\circ K$ cosmic microwave background (CMB) serves as the best evidence of this overall background.

Moreover, this overall background sees the lepton world, or the various atomic worlds. The couplings are all dimensionless in the 4-dimensional Minkowski space-time.

And, moreover, this overall background sees the quark world, or the various nuclear worlds. Again, the couplings are all dimensionless in the 4-dimensional Minkowski space-time.

These "dimensionless-ness" should enter our discussions of ultraviolet divergences. But we don't know how - maybe we anticipate to have a good starting point to construct the Standard Model of All Centuries.

Why do we need the "ignition" channel? Why is the ignition channel not the SM Higgs η ? We set out to use the SM Higgs as the "ignition" channel, but soon realized that the "ignition" channel with the purely family η' works out "perfectly". In fact, the first version of [2] was based on that the ignition channel was the SM Higgs η - the standard wisdom. As for why we need the "ignition" channel, we still need a good answer. (We believe that the God would not choose more than one ignition point, in this game.)

Note that $\theta_P = 45^\circ$ corresponds to the situation that $\Phi(3, 2)$ be equally divided by $\Phi(1, 2)$ (SM Higgs) and $\Phi(3, 1)$ (purely family). The situation corresponds to that it is not yet "ignited" ($\mu_2^2 = 0$). How do these translate into the temperature situations (via the Big Bang or via big inflation)? The elusive purely family Higgs η' as the "ignition" channel gives us a few interesting questions.

6 Concluding Remarks

To close this paper, we append a few remarks just to remind ourselves the leading physical issues that we may pursue after.

To verify this Standard Model is the experimental search for the family Higgs η'_1 , or $\eta'_{2,3}$, or charged family Higgs ϕ_1^+ and $\phi_{2,3}^+$, or pure family Higgs η' , in a proposed 120 GeV μ^+e^- collider [8].

The major implication of the family gauge theory is in fact a multi-GeV or sub-sub-fermi gauge theory (a new force field of a few 10^{-15} cm in the range), assuming that the ordering in the coupling constants, $g_W/g_c \sim \kappa/g_W$, is reasonable. Note that the lepton world are shielded from this $SU_f(3)$ theory against the QED Landau's ghost, and similarly the quark world from strong-interaction $SU_c(3)$. The $g-2$ anomaly certainly deserves another serious look in this context [9].

In this Standard Model, the masses of quarks are diagonal, or the singlets in the $SU_f(3)$ space; those of the three charged leptons are $m_0 + a\lambda_2 + b\lambda_5 + c\lambda_7$ (before diagonalization) and the masses of neutrinos are purely off-diagonal, i.e. $a'\lambda_2 + b'\lambda_5 + c'\lambda_7$. This result is very interesting and very intriguing.

This result follows from the above curl-dot product, or, the $\epsilon^{abc}\bar{\Psi}_{L,a}\Psi_{R,b}\Phi_c$ product, i.e. the $SU_f(3)$ operation, in writing the coupling(s) to the right-handed lepton triplets.

In addition, neutrinos oscillate among themselves, giving rise to a lepton-flavor-violating interaction (LFV). There are other oscillation stories, such as the oscillation in the $K^0 - \bar{K}^0$ system, but there is a fundamental "intrinsic" difference here - the $K^0 - \bar{K}^0$ system is composite while neutrinos are "point-like" Dirac particles. We have standard Feynman diagrams for the kaon oscillations but similar diagrams do not exist for point-like neutrino oscillations - our Standard Model solves the problem, maybe in a unique way.

Thinking it through, it is true that neutrino masses and neutrino oscillations may be regarded as one of the most important experimental facts over the last thirty years [10].

In fact, certain LFV processes such as $\mu \rightarrow e + \gamma$ [10], $\mu + A \rightarrow A^* + e$, etc., are closely related to the most cited picture of neutrino oscillations [10]. In our previous publications [11], it was pointed out that the cross-generation or off-diagonal neutrino-Higgs interaction may serve as the detailed mechanism of neutrino oscillations, with some vacuum expectation values of the family Higgs $\Phi^0(3, 2)$. So, even though we haven't seen, directly, the family gauge bosons and family Higgs particles, we have already seen the manifestations of their vacuum expectation values.

Moreover, in this Standard Model, neutrinos and antineutrinos are the *only long-lived* dark-matter particles [12]. Cosmic background ν 's from the early Universe, owing to the nonzero neutrino masses, would cluster in lumps, in neutrino halos, five times in weight the visual objects, such as stars, planets, etc. Since the neutrino halo is incompressible (as a

Fermi gas), this would prevent the final collapse of the visual ordinary-matter object into a black hole [13].

Thus, the Standard Model, i.e., the quantum 4-dimensional Minkowski space-time with the force-fields gauge-group structure $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ built-in from the very beginning, we understand our Universe; that is, all the dark-matter particles and all the ordinary-matter particles are accounted for. Formation of the black holes for visual ordinary-matter objects are stopped by the neutrino halos (of the five times in weight). Our Standard Model gives us the perfect Universe which we are living.

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